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A STUDY OF THE TIME VARIATION OF  
PERTURBATION KINETIC ENERGY AT  
THE 500-MILLIBAR LEVEL

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by

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Submitted in partial fulfillment of  
the requirements for the degree of  
MASTER OF SCIENCE  
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A STUDY of the TIME VARIATION of PERTURBATION KINETIC ENERGY  
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from the

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## ABSTRACT

The transfer of kinetic energy by individual wave numbers is computed at various latitudes using data obtained from a harmonic analysis of the hemispheric 500 millibar pressure height. Linear correlations are sought between the computed values of kinetic energy transfer and computed 24-hour changes of kinetic energy of the perturbations for the same and for adjacent wave numbers.

The writer wishes to express sincerest appreciation to Professor Frank L. Martin for his suggestion of the topic and for his guidance during the study.

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# LIST OF SYMBOLS

Symbol	Definition
$\lambda$	longitude
$\phi$	latitude
$p$	pressure
$a$	radius of earth
$f$	coriolis parameter
$g$	acceleration of gravity
$\Omega$	angular momentum of earth
$\mathbf{V}$	vector wind
$u$	eastward wind component
$v$	northward wind component
$w$	vertical component
$(\bar{\quad})$	longitudinal mean of quantity
$(\quad)'$	perturbation quantity
$(\quad)_g$	geostrophic component
$F$	friction force
$\rho$	air density
$z$	contour height
$n$	wave number
$A(n)$	amplitude, perturbation contour height
$\Psi(n)$	phase, perturbation contour height
$H(n)$	amplitude of $u$ and $v$ Fourier analyses at wave $n$
$\sigma(n)$	phase of $u$ and $v$ Fourier analyses at wave $n$
$\chi$	phase difference between two $n$ -th harmonics of $u$ and $v$
$C_{uv}(n)$	cospectrum of $u$ and $v$ at wave number $n$
$P_u(n)$ or $P_v(n)$	power spectrum of $u$ or $v$ at wave number $n$
$r$	linear correlation coefficient

## 1. Introduction.

One phase in the study of the general circulation of the atmosphere has been the investigation of mechanisms by which the kinetic energy balance is maintained in spite of continual frictional dissipation. Widger [14], Kao [6], and others have established that a mean northward transport of kinetic energy in mid-latitudes is effected by the large scale perturbations in the atmosphere. Saltzman [9, 10] has investigated the spectral distribution of perturbation kinetic energy. Saltzman and Fleisher [11] have studied the rate of transfer of kinetic energy between the perturbations and the mean flow on a daily basis, while Benton and Kahn [1] have conducted a similar investigation on a seasonal basis. Saltzman and Fleisher [12] have also estimated the interchange of eddy kinetic energy between groups of perturbations, assuming that the total kinetic energy interchange is zero.

It is the purpose of this study to correlate the horizontal geostrophic transfer of kinetic energy by wave number with 24-hour changes of kinetic energy of the same and different wave numbers existing in the same and nearby latitude belts. The data used are from a Fourier analysis of 500-mb contours at successive 5-degree latitude belts. This Fourier analysis, which covers the winter season of 1947-48, was made available by the U. S. Navy Weather Research Facility, Norfolk. Further discussion concerning the data appears in a later section.

## 2. The energy transfer function.

Using the equations of quasi-horizontal motion, Saltzman [9] has developed an expression for the change of eddy kinetic energy within a closed mass of air,  $M$ , e.g. the entire atmosphere, which is

$$\begin{aligned} \frac{\partial}{\partial t} \int_M KE' \delta m = & - \int_M \left[ \overline{u'v'} \frac{\cos \phi}{a} \frac{\partial}{\partial \phi} \left( \frac{\bar{u}}{\cos \phi} \right) + \overline{v'v'} \frac{1}{a} \frac{\partial \bar{v}}{\partial \phi} \right. \\ & - \overline{u'u'} \frac{\bar{v}}{a} \tan \phi + \overline{u'\omega'} \frac{\partial \bar{u}}{\partial p} + \overline{v'\omega'} \frac{\partial \bar{v}}{\partial p} \left. \right] \delta m \\ & - \int_M g \overline{V' \cdot \nabla Z'} \delta m - \int_M \overline{V' \cdot F} \delta m \end{aligned} \quad (2.1)$$

Here  $\phi$  is latitude,  $\lambda$  longitude,  $p$  pressure, and  $a$ , the earth-radius. The eastward and northward components of the wind are  $u$  and  $v$  respectively, while the vertical motion is  $\omega$ . Primes are associated with perturbation quantities and bars with averages around a latitude circle. Thus, for example,

$$\overline{(\quad)} = \frac{1}{2\pi} \int_0^{2\pi} (\quad) \delta \lambda$$

indicates the barred-type average. The mass element  $\delta m$  in equation (2.1) will be discussed in a later section.

Saltzman also shows that the change in the mean kinetic energy,  $\frac{\partial}{\partial t} \int_M \overline{KE} \delta m$ , contains the first integral on the right in (2.1), but with the opposite sign affixed.

It may be seen that the first integral in equation (2.1) contains terms which are products of the macroscopic Reynolds stresses and the derivatives of the mean flow.

The remaining integrals represent contributions to the eddy kinetic energy by conversion of "eddy available potential energy" and frictional dissipation, respectively. Inasmuch as this study is concerned only with the effects of horizontal transfer of kinetic energy, no further discussion of the geopotential or frictional terms will be entertained.

If the geostrophic approximation is made, as is done in this study, the mean meridional component  $\bar{v}$  will vanish. Furthermore, since it is impossible to determine vertical motions from the Fourier analysis of the height field at a single level, those terms involving  $\omega'$  will be neglected.

With these simplifications, the eddy kinetic energy change may be expressed in the form

$$\frac{\partial}{\partial t} \int_M KE \delta m \doteq T^* = - \int_M \overline{u'v'} \frac{\cos \phi}{a} \frac{\partial}{\partial \phi} \left( \frac{\bar{u}}{\cos \phi} \right) \delta m \quad (2.2)$$

For an interpretation of the integral appearing in equation (2.2), a vertical element of area,  $\delta s = a \cos \phi \delta \lambda \delta z$ , contained in a latitude wall, will first be considered. Here the eastward momentum per unit mass of air is

$$u = \bar{u} + u'$$

Any transport of eastward momentum normal to  $\delta s$  must be brought about by the meridional wind component

$$v = \bar{v} + v'$$

With the geostrophic approximation, the longitudinally-averaged meridional transport of eastward momentum due to large scale turbulence is given, after Haltiner and

Martin [4],

$$\overline{u'v'} = \overline{(u_g - u_g)(v_g - v_g)} = \overline{u_g v_g}$$

The integral in equation (2.2) will be denoted  $T^*$ , and will be termed the "eddy transfer function." It will eventually involve contributions to the kinetic energy change by different wave numbers. A positive value for  $T^*$  represents an exchange of kinetic energy from the mean flow into that of the eddy flow.

### 3. The energy-transfer spectrum.

Since  $u$  and  $v$  may be considered real and continuous functions of longitude  $\lambda$ , each having unique values at all points of a given latitude circle, they may be expanded into Fourier series of the form

$$u(\lambda) = a_u(0) + \sum_{n=1}^{\infty} a_u(n) \cos n\lambda + b_u(n) \sin n\lambda \quad (3.1)$$

where

$$\begin{aligned} a_u(0) &= \frac{1}{2\pi} \int_0^{2\pi} u(\lambda) d\lambda \\ a_u(n) &= \frac{1}{\pi} \int_0^{2\pi} u(\lambda) \cos n\lambda d\lambda \\ b_u(n) &= \frac{1}{\pi} \int_0^{2\pi} u(\lambda) \sin n\lambda d\lambda \end{aligned} \quad (3.2)$$

Likewise

$$v(\lambda) = a_v(0) + \sum_{n=1}^{\infty} a_v(n) \cos n\lambda + b_v(n) \sin n\lambda \quad (3.3)$$

where  $a_v(0)$ ,  $a_v(n)$  and  $b_v(n)$  are defined similarly to equations (3.2). Equations (3.1) and (3.3) may also be

expressed, in terms of amplitude and phase angle, as

$$u(\lambda) = a_u(0) + \sum_{n=1}^{\infty} H_u(n) \sin n(\lambda - \sigma) \quad (3.4a)$$

$$v(\lambda) = a_v(0) + \sum_{n=1}^{\infty} H_v(n) \sin n(\lambda - \sigma) \quad (3.4b)$$

with the amplitude and phase angle of equation (3.4a) given by

$$H_u(n) = \sqrt{a_u(\lambda)^2 + b_u(\lambda)^2} \quad \text{and} \quad n\sigma_u(n) = \tan^{-1} \left[ -\frac{a_u(n)}{b_u(n)} \right] \quad (3.5)$$

Now the copower series of  $u$  and  $v$  may be defined, after Kahn [5],

$$\frac{1}{2\pi} \int u(\lambda) v(\lambda) \delta\lambda = a_u(0) a_v(0) + \sum_{n=1}^{\infty} C_{uv}(n) \quad (3.6)$$

where it may be verified upon combining equations (3.4) and (3.6), that the copower spectrum, or cospectrum, has the form

$$C_{uv}(n) = \frac{1}{2} H_u(n) H_v(n) \cos n(\sigma_u(n) - \sigma_v(n)) \quad (3.7)$$

Here the symbols  $H(n)$ , and  $\sigma(n)$ , are defined as in equations (3.5). It may also be noted that for the latitudinal-mean values of  $u^2(\lambda)$  or  $v^2(\lambda)$ , the cospectrum  $C_{uv}$  in equation (3.6), becomes simply the power spectrum of  $\overline{u^2(\lambda)}$  or  $\overline{v^2(\lambda)}$

$$P_u(n) = \frac{1}{2} H_u^2(n) \quad \text{or} \quad P_v(n) = \frac{1}{2} H_v^2(n) \quad (3.8)$$

Now the value of  $C_{uv}$  at any given wave number may be shown to be independent of its values at any other wave number.

Equations (3.7) and (3.8) may therefore be used to obtain the contribution of a specific wave number to either the cospectrum or the power spectrum, as the case may be.

A useful expression for  $\overline{u'v'}$  may be obtained by subtracting  $a_u(0) a_v(0)$  from the left side of equation (3.6), after which

$$u'v' = \sum_{n=1}^{\infty} C_{uv}(n)$$

Making this substitution into the transfer equation (2.2) gives

$$T^* = - \int_M \sum_{n=1}^{\infty} C(n) \frac{\cos \phi}{a} \frac{\partial}{\partial \phi} \left( \frac{\bar{u}}{\cos \phi} \right) = \sum_{n=1}^{\infty} T(n)^* \quad (3.9)$$

Thus  $T^*$  may be expressed as a sum of contributions for each wave number.

A more convenient form of equation (3.9) will now be derived. Making use of the geostrophic approximation to the horizontal flow, the eastward and northward components are

$$\begin{aligned} u_g &= - \frac{g}{fa} \frac{\partial Z}{\partial \phi} \\ v_g &= \frac{g}{f a \cos \phi} \frac{\partial Z}{\partial \lambda} \end{aligned} \quad (3.10)$$

Here  $g$  is the acceleration of gravity,  $f$  the coriolis parameter, and  $Z$  contour height. The other symbols are as previously defined. Denoting the  $n$ -th harmonic of

the height field at latitude  $\phi$  as

$$Z(n) = A(n) \cos n(\lambda - \Psi(n)) \quad (3.11)$$

where  $A(n)$  and  $\Psi(n)$  are the amplitude and phase, respectively, at latitude  $\phi$ , equations (3.10) are expressible as

$$u(n) = -\frac{g}{fa} \left[ \frac{\partial A(n)}{\partial \phi} \cos n(\lambda - \Psi(n)) + nA(n) \frac{\partial \Psi(n)}{\partial \phi} \sin n(\lambda - \Psi(n)) \right] \quad (3.12)$$

$$v(n) = -\frac{gnA(n)}{fa \cos \phi} \sin n(\lambda - \Psi(n)) \quad (3.13)$$

Henceforth the wave number  $n$  will be understood in  $A(n)$  and  $\Psi(n)$ . Equation (3.12) may also be written in the form

$$u_g(n) = -\frac{g}{fa} \left[ \sqrt{\left( \frac{\partial A}{\partial \phi} \right)^2 + \left( nA \frac{\partial \Psi}{\partial \phi} \right)^2} \right] \sin [n(\lambda - \Psi) + \chi] \quad (3.14)$$

where the angle  $\chi$  is given by

$$\chi = \tan^{-1} \left( \frac{\partial A}{\partial \phi} / nA \frac{\partial \Psi}{\partial \phi} \right) \quad (3.15)$$

and is therefore the phase difference between  $u_g$  and  $v_g$ . Using the expressions (3.13), (3.14), (3.15) and (3.7), equation (3.9) takes the form

$$T_{(n)}^* = -\frac{g}{2a^2} \int_m \left[ \left( \frac{n^2 A^2 \left( \frac{\partial \Psi}{\partial \phi} \right)}{f^2 \cos \phi} \right) \frac{\cos \phi}{a} \frac{\partial}{\partial \phi} \left( \frac{\bar{u}}{\cos \phi} \right) \right] \delta m \quad (3.16)$$

Equation (3.16) will be termed the energy transfer spectrum. By simplifying the quantity within the square brackets, the contribution for a single wave number becomes

$$T_{(n)}^* = -\frac{g}{2a^2} \left[ \left( \frac{n^2 A^2 \left( \frac{\partial \Psi}{\partial \phi} \right)}{a f^2 \cos \phi} \right) \left( \left( \frac{\partial \bar{u}}{\partial \phi} \right) + \bar{u} \tan \phi \right) \right] \delta m \quad (3.17)$$

#### 4. The 24-hour change of eddy kinetic energy.

An approximation to the change of eddy kinetic energy in a closed system resulting from all of the processes of equation (2.1) is readily made. The eddy kinetic energy per unit mass at any position  $(\lambda, \phi)$  is given by

$$KE(n) = \frac{V(n)^2}{2} = \left( \frac{u^2(n) + v^2(n)}{2} \right) \quad (4.1)$$

Using equations (3.8), (3.12) and (3.13), one obtains the latitudinally-averaged or spectral values at latitude  $\phi$

$$\begin{aligned} u^2(n) &= P_{u^2}(n) = \frac{g^2}{2f^2a^2} \left[ \left( \frac{\partial A}{\partial \phi} \right)^2 + \left( nA \frac{\partial \psi}{\partial \phi} \right)^2 \right] \\ v^2(n) &= P_{v^2}(n) = \frac{g^2}{2f^2a^2} \left( \frac{nA}{\cos \phi} \right)^2 \end{aligned} \quad (4.2)$$

Substitution of equations (4.2) into (4.1) yields the zonally-averaged geostrophic eddy kinetic energy per unit mass. For a closed system of mass  $M$ ,

$$KE(n) = \frac{g}{4a^2} \int \frac{1}{f} \left[ \left( \frac{\partial A}{\partial \phi} \right)^2 + \left( nA \frac{\partial \psi}{\partial \phi} \right)^2 + \left( \frac{nA}{\cos \phi} \right)^2 \right] \delta m \quad (4.3)$$

The mean 24-hour change is found using the simple difference, shown symbolically,

$$\Delta KE(n) = KE(n)_{\text{day } i+1} - KE(n)_{\text{day } i} \quad (4.4)$$

#### 5. Finite-difference forms.

The element of mass  $\delta m$  of equations (3.17) and (4.3) may be expressed initially as

$$\delta m = \rho a^2 \cos \phi \delta \lambda \delta \phi \delta z \quad (5.1)$$

which, upon substitution from the hydrostatic equation, becomes

$$\delta m = \frac{a^2 \cos \phi}{g} \delta \lambda \delta \phi \delta p \quad (5.2)$$

where  $\delta p$ , the pressure increment is considered positive.

In a series of papers, Saltzman [10], and Saltzman and Fleisher [11, 12] have been careful to simulate a closed system by using a large portion of the northern hemisphere at 500 mb, for example, from  $15^\circ$  N to  $80^\circ$  N. In this thesis, it seemed advantageous, from the point of view of a possible forecast tool, to investigate kinetic-energy changes in ten-degree latitude belts, even though such belts may not constitute closed systems.

The major purpose of this study is to investigate possible relationships between  $T'_{(n)}^*$  and  $\Delta KE_{(n)}$  at the 500-mb level. Combining equations (3.17) and (5.2) leads to

$$T'_{(n)}^* = -\frac{g}{8a^2} \int_{\phi-\Delta\phi}^{\phi+\Delta\phi} \int_{p_1}^{p_2} \int_0^{2\pi} \left( \frac{n^2 A^2 \left( \frac{\partial \psi}{\partial \phi} \right)}{\sin^2 \phi} \right) \left( \frac{\partial \bar{u}}{\partial \phi} + \bar{u} \tan \phi \right) \delta \lambda \delta \phi \delta p \quad (5.3)$$

Integration of (5.3) with respect to longitude will increase the constant term by a factor of  $2\pi$ . The remaining integrations are with respect to pressure, where

$$|p_2 - p_1| = \Delta p = 1 \text{ mb}$$

centered at 500 mb, and with respect to latitude, where  $2\Delta\phi = 10$  degrees, or  $\pi/18$  radians.

Employing finite differences and the geostrophic approximation, the quantity  $\frac{\partial \bar{u}}{\partial \phi}$  in equation (5.3) becomes

$$\frac{\partial \bar{u}}{\partial \phi} = - \frac{g}{2 \Omega a \Delta \phi} \left[ \frac{(\bar{Z}_{\phi+5} - \bar{Z}_{\phi})}{\sin(\phi+2.5)} - \frac{(\bar{Z}_{\phi} - \bar{Z}_{\phi-5})}{\sin(\phi-2.5)} \right] \quad (5.4)$$

where  $\bar{Z}$  is the height of the mean height contour at the latitude indicated by the subscripts. Moreover  $\bar{u}$ , at the central latitude  $\phi$ , may be expressed as the arithmetic mean

$$\bar{u} = - \frac{g}{2 \Omega a \Delta \phi} \left[ \frac{(\bar{Z}_{\phi+5} - \bar{Z}_{\phi})}{\sin(\phi+2.5)} + \frac{(\bar{Z}_{\phi} - \bar{Z}_{\phi-5})}{\sin(\phi-2.5)} \right] \quad (5.5)$$

Substituting (5.4) and (5.5) into the integrated form of equation (5.3), the finite difference form may be stated as

$$T(n)^* = \frac{\pi g^2 n^2 A^2}{4 a^2 \Omega^2 \sin^2 \phi} \left( \frac{\psi_{\phi+5} - \psi_{\phi-5}}{2 \Delta \phi} \right) \left\{ \left[ \frac{(\bar{Z}_{\phi+5} - \bar{Z}_{\phi})}{\sin(\phi+2.5)} - \frac{(\bar{Z}_{\phi} - \bar{Z}_{\phi-5})}{\sin(\phi-2.5)} \right] \right. \\ \left. + \left[ \frac{(\bar{Z}_{\phi+5} - \bar{Z}_{\phi})}{\sin(\phi+2.5)} + \frac{(\bar{Z}_{\phi} - \bar{Z}_{\phi-5})}{\sin(\phi-2.5)} \right] \right\} \quad (5.6)$$

Combining equations (4.3) and (5.2) yields

$$KE(n) = \frac{g}{4 (2 \Omega)^2} \int_{\phi-\Delta \phi}^{\phi+\Delta \phi} \int_{p_1}^{p_2} \int_0^{2\pi} \left[ \frac{1}{\sin^2 \phi} \left( \frac{\partial A}{\partial \phi} \right)^2 + \left( n A \frac{\partial \psi}{\partial \phi} \right)^2 + \left( \frac{n A}{\cos \phi} \right)^2 \right] \delta \lambda \delta p \delta \phi \quad (5.7)$$

which, when integrated as before, takes the finite-difference form

$$KE(n) = \frac{g \pi}{8 \Omega^2} \left\{ \frac{\cos \phi}{\sin^2 \phi} \left[ \left( \frac{A_{\phi+5} - A_{\phi-5}}{2 \Delta \phi} \right)^2 + n^2 \bar{A}^2 \left( \frac{\psi_{\phi+5} - \psi_{\phi-5}}{2 \Delta \phi} \right)^2 \right. \right. \\ \left. \left. + \left( \frac{n \bar{A}}{\cos \phi} \right)^2 \right] \right\} 2 \Delta \phi \quad (5.8)$$

where

$$\bar{A} = \frac{1}{4} (A_{\phi+5} + 2A_{\phi} + A_{\phi-5}), \text{ and } 2 \Delta \phi = \pi/15$$

It should be noted that the same integrations have been performed in arriving at equation (5.8) as for (5.6). The

units of  $KE(n)$  are therefore those of kinetic energy per mb and per  $10^\circ$  latitude belt.

#### 6. Data processing and computation.

Data for the evaluation of equations (5.6) and (5.8) were taken from a Fourier analysis of the 500-mb pressure surface which was made by the Naval Weather Research Facility [13]. The Facility used the Historical Series 500-mb hemispheric charts for the winter season of 1947-48. Mean 500-mb heights, and the amplitude and phase for each of the first 18 wave numbers were computed. Values were available for five-degree intervals from 15 degrees to 70 degrees North latitude. Data selected for this study were for the 33-day period from 31 December through 1 February.

Because of time limitations, only waves 1 through 12 were considered. However, results of Duggan [3] and Melhorn and Le Dew [7] indicate that contributions from wave numbers higher than ten may be expected to be relatively small.

In the case of phase-angle data, further refinement was necessary. Phase angles were given in terms of distance from the prime meridian to the ridge line of the wave in question. Because of the method of tabulation of phase angles used in the original harmonic analysis, this value did not always pertain to the ridge nearest the meridian. Thus, if the phase difference obtained from the reported values was in excess of one-half wave length, it

was necessary to subtract one wave length from the original difference. Fig. 1 schematically depicts this process of identification.

Computation of equations (5.6) and (5.8) was performed for each of five ten-degree latitude belts, centered respectively at 65, 55, 45, 35, and 25 degrees. The Control Data Corporation CDC 1604 computer was employed in these evaluations, being ideally suited because of its high speed and large capacity.

## 7. Results and conclusions.

It was indicated previously that if integration of the transfer equation (2.2) is performed over the entire atmosphere, an approximate measure of the transfer of kinetic energy between the eddies and the mean zonal wind is obtained. From the evaluation of this integral over a 10-degree latitude belt, as is done in this study, only a partial verification of this approximation can be inferred. Fig. 2 shows the 33-day mean-energy transfer spectrum  $T(n)^*$  at the central latitude of each 10 degree latitude belt. A positive value of  $T(n)^*$  indicates a contribution to the kinetic energy of the eddy flow from the mean wind, whereas a negative value indicates that the eddies supply kinetic energy to the mean wind.

Perhaps the most striking feature of the spectra shown in Fig. 2 is the change in sign of the transfer between 35°N and 45°N. At the lower latitudes, transfer from the eddies to the mean flow is indicated. This is in keeping with the findings of Kao [6] who shows a maximum

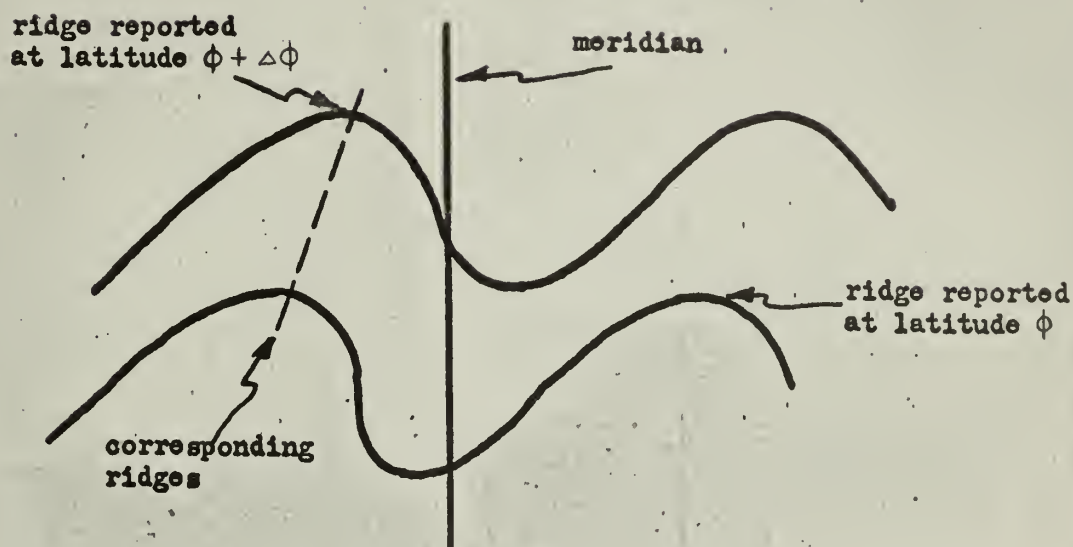


Figure 1. Schematic illustration of the identification of corresponding wave pairs at neighboring latitudes.

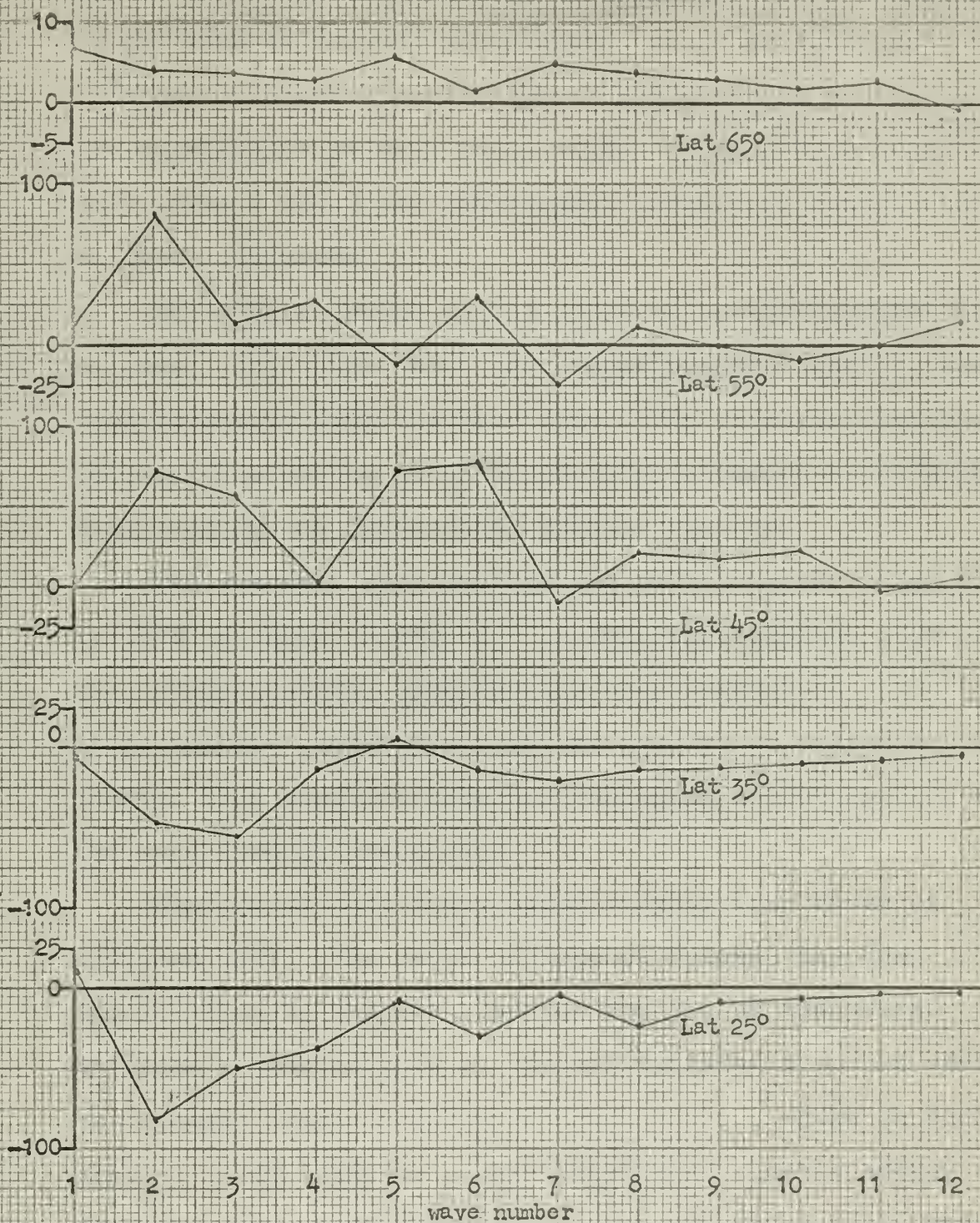


Figure 2. 33-day mean kinetic energy spectra by latitude belt for the period 31 December 1947 through 1 February 1948. Ordinate values are for the central latitude of each belt shown. Units are  $(\text{m}^2/\text{sec}^3/\text{nb}) \times 10^{-8}$  per ten-degree latitude belt.

convergence of northward transport of kinetic energy at 500 mb up to approximately  $36^{\circ}\text{N}$ , which is in the vicinity of the zone of maximum westerlies. Thus, at latitude  $25^{\circ}\text{N}$  and  $35^{\circ}\text{N}$  the transfer spectrum appears to assist in the maintenance of mean zonal winds. It should be noted that waves 2 and 3 are the largest individual contributors at these latitudes.

At  $45^{\circ}\text{N}$ , a change in the sense of energy transfer may be seen. All waves, with the exceptions of 7 and 11, now receive energy from the mean flow. The transfer from mean to eddy flow occurs at  $55^{\circ}\text{N}$  and  $65^{\circ}\text{N}$ , although at  $65^{\circ}\text{N}$  the magnitude of the exchange is greatly reduced.

The spectral estimate for the full latitude range  $20^{\circ}\text{N}$  to  $70^{\circ}\text{N}$  is shown in Fig. 3. In this instance the sign of the energy transfer spectrum has been changed to  $-T'(\omega)$  to facilitate comparison with the results of Saltzman [10]. In Fig. 3, a positive sign now indicates a contribution to the mean wind by the eddies.

Results of this study agree roughly with those of Saltzman [10]. The major difference between the two spectra lies in the net transfer for all of the waves. Saltzman's results show an overall transfer from the eddies to the mean flow, [i.e.,  $\left(\sum_{n=1}^{12} -T'(\omega_n)\right) > 0$ ], while in the present study there is a net transfer of kinetic energy from the mean to the eddy flow,  $\left[\sum -T'(\omega_n) < 0\right]$ . This is not unexpected in the light of Saltzman and Fleisher's [11] quoted values of the standard deviation of the transfer, which, in general, exceed the value of the spectral estimate.

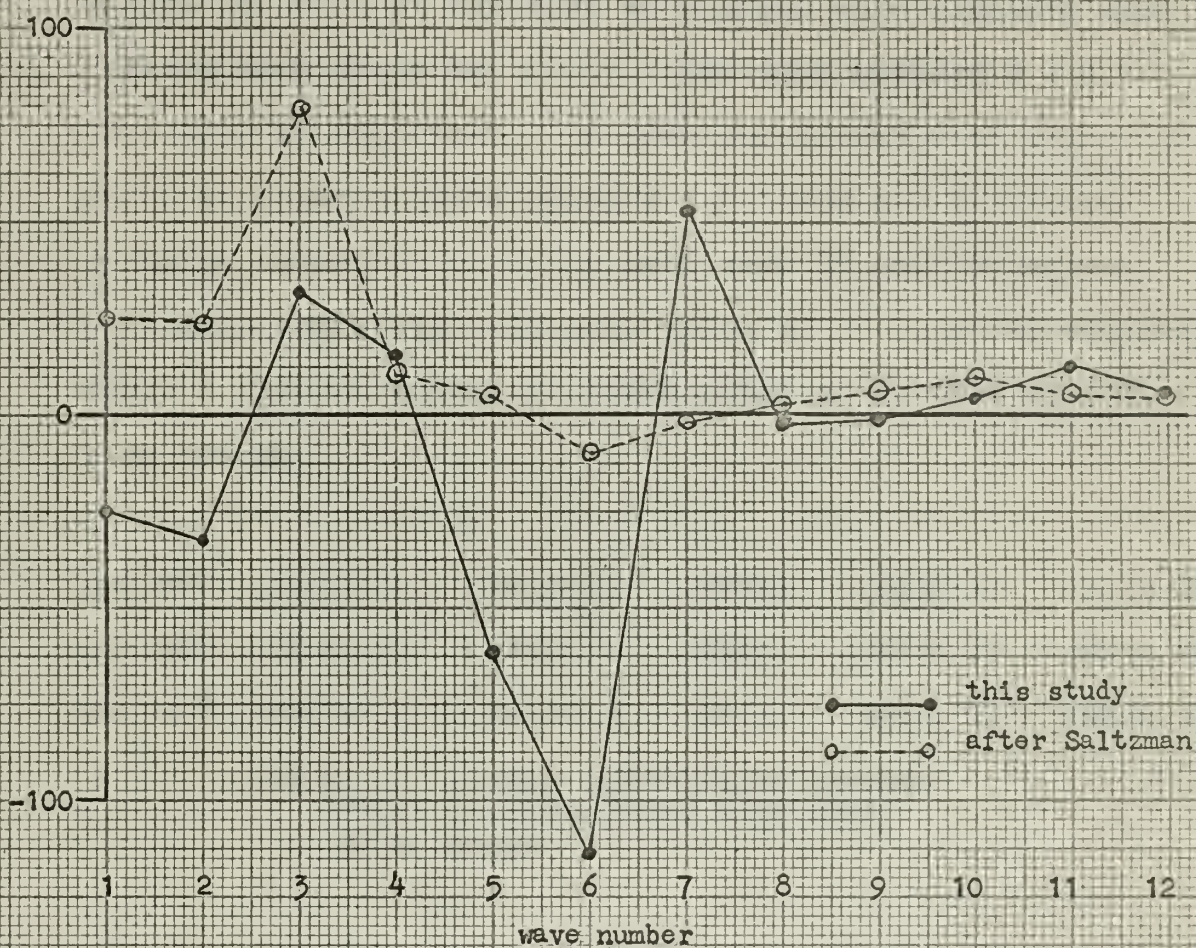


Figure 3. 33-day mean kinetic energy spectrum for the entire latitude range  $20^{\circ}\text{N}$  to  $70^{\circ}\text{N}$  for the period 31 December 1947 through 1 February 1948. Units are as in Fig.2. A positive value indicates a contribution to the mean energy from the eddy energy. Dashed curve is after Saltzman [a] for the month of January 1949.

It is reasonable to suppose that changes in  $T(n)^*$  at a given wave number may be accompanied by changes in the kinetic energy associated with other wave numbers at the same latitude and at adjacent latitudes. Accordingly, values of  $T(n)^*$  at latitude 45 degrees were linearly correlated with those of  $\Delta KE$  for wave numbers  $n = 1, 2, \dots, 12$ . In addition,  $T(n)^*$  was correlated with values of  $\Delta KE$  at adjacent wave numbers  $n \pm 1$  and  $n \pm 2$ . Corresponding values of  $\Delta KE$  at latitudes 35 and 55 degrees were also correlated with the value of  $T(n)^*$  at 45 degrees. Table 1 gives a schematic representation of this correlation system.

Lat 55°	$\Delta KE(n-2)$	$\Delta KE(n-1)$	$\Delta KE(n)$	$\Delta KE(n+1)$	$\Delta KE(n+2)$
Lat 45°	$\Delta KE(n-2)$	$\Delta KE(n-1)$	$\Delta KE(n)$	$\Delta KE(n+1)$	$\Delta KE(n+2)$
Lat 35°	$\Delta KE(n-2)$	$\Delta KE(n-1)$	$\Delta KE(n)$	$\Delta KE(n+1)$	$\Delta KE(n+2)$

Table 1. 15-way correlation scheme.  
 $T(n)^*$  is correlated with each  
 $\Delta K.E.$  shown.

Table 2 shows the results of the 15-way correlation system in which  $T(n)^*$  at 45 degrees is correlated with each of the 15 sets of values of  $\Delta KE(n, n \pm 1, n \pm 2)$  at latitude 35, 45, and 55 degrees. The values of  $\Delta KE$  are centered 12 hours prior to the time of  $T(n)^*$ . Tables 3, 4, and 5 show the results with  $\Delta KE$  centered 12, 36, and 60 hours respectively after the time of  $T(n)^*$ . The tables are arranged in the schematic fashion shown in Table 1.

An excellent statistical routine, written for the CDC 1604 computer by Campbell [2], was employed in the computation of the correlations.

In proceeding to the correlation analysis, it was necessary to establish confidence limits for the correlation coefficient. The number of independent pairs in all the correlations was 30. According to the z-test, as given by Panofsky and Brier [8], a correlation coefficient of .30 is significant at the 90% level of belief.

The physical principle to be tested is the conversion of the transfer-energy by wave number  $n$  into the kinetic energy of wave number  $n$ ,  $n \pm 1$ ,  $n \pm 2$ . Saltzman [9, equation (49)] has derived a transform of equation (2.1). His result includes all of the terms which appear in (2.1), in spectral form, and in addition, terms representing interaction of kinetic energy from wave number  $n$  to all other waves. In a later paper, Saltzman and Fleisher [12] have estimated the rate of energy interaction at 500 mb. They have shown that energy transitions tend to occur between a middle group of waves, ( $n = 6$  through 10) called "cyclone waves," into the longer wave group ( $n = 1$  through 5) and the short wave group ( $n = 11$  through 15). The present study is more limited in scope, in that the range of waves considered is only  $(n - 2)$  through  $(n + 2)$ . On the other hand, investigation by means of correlations between specific wave numbers may make available more detailed criteria for the energy exchange between waves at latitudes  $35^\circ$ ,  $45^\circ$ , and  $55^\circ$ , in terms of  $\hat{T}_{nm}$  at 45 degrees latitude.

For the purpose of such an investigation correlation coefficients from a particular row and column of each of Tables 2, 3, 4, and 5 were selected in accordance with the criteria that each set of four successive coefficients contain at least:

- or (a) two significant correlations
- (b) one significant correlation together with a 24-hour or 48-hour difference in correlation of  $\pm .60$ .

All together there were 47 sets of successive correlations in Tables 2 through 5 which satisfied criteria (a) or (b).

As an example, if the row and column just mentioned correspond to (11, 9), each of the four tables would be entered with the central n-value,  $n = 11$ , and column  $n - 2 = 9$ . This particular correlation coefficient would be symbolized  $\bar{r}_{11,9}$ . If a row corresponding to the higher latitude ( $55^{\circ}\text{N}$ ) is selected, a bar is placed over the  $r$  (e.g.,  $\bar{r}$ ), whereas if the lowest latitude is used, a bar is placed below the  $r$  (e.g.,  $\underline{r}$ ). For a choice of a central latitude, no bar is employed.

For example,  $\underline{r}_{11,9}$  signifies the correlation between  $T^*(11)$  computed at  $45^{\circ}\text{N}$  versus  $\Delta\text{KE}(9)$  computed at latitude 35 degrees. Of this particular set of four correlations, three were significant at the 90% level or higher. The set of four is plotted in Fig. 4a.  $\bar{r}_{11,11}$  shows a similar pattern, but in this case has a significant positive correlation at time  $t - 12$  hours. This initially positive significant correlation tends to confirm the hypothesis that an above-normal value of  $T^*(n)$  is associated with an

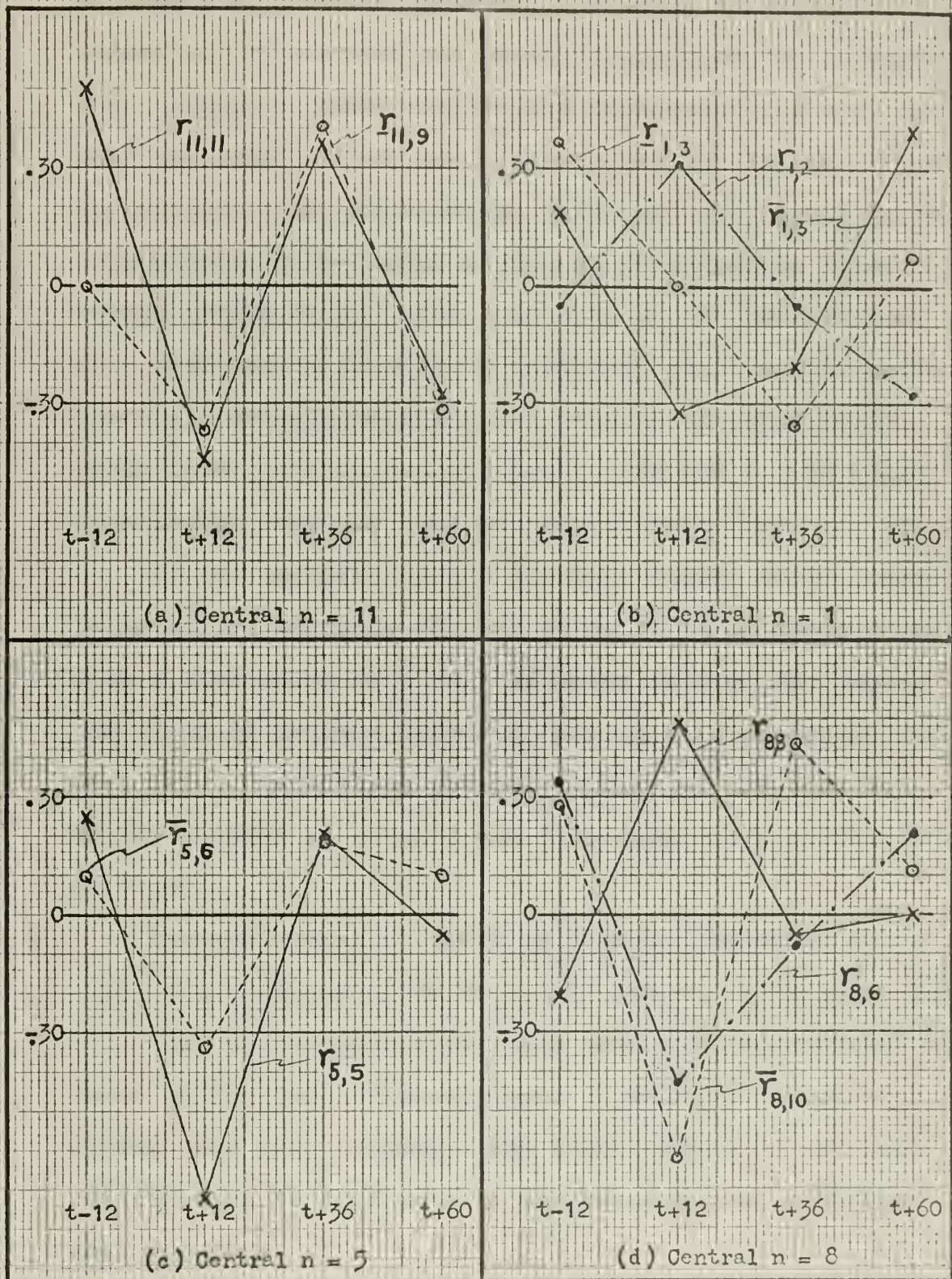


Figure 4. Sequential correlation coefficient sets of  $T^*(n)$  with  $\Delta KE$  for selected central  $n$ -values. Times are in hours from the computation time  $t$  of  $T^*(n)$ . Values of  $|r| > .30$  are significant at the 90% level of belief.

above-normal value in  $\Delta KE(n)$  at the same wave number. At time  $t + 12$  hours, however, the value is negative, oscillating to positive and negative again on the succeeding two days. These results are consistent for all correlation sets meeting the criteria (a) or (b) in  $n \geq 10$  and suggest that above-normal value of  $T(n)^*$  tends to be followed by below-normal change in perturbation kinetic energy twelve hours later. Several of the correlation sets examined in this group displayed this two-day periodicity. Certain of the other sets in the short-wave group lose significance at time  $t + 36$  and  $t + 60$  hours even though they may have met the criteria (a) or (b).

At the opposite end of the wave number scale, there is again the tendency for  $\Delta KE$  (2 or 3) to be above-normal with a high value of  $T(n)^*$ , as shown in Fig. 4b. Again  $\Delta KE$  (2 or 3) becomes negative, but in this instance, two days later, indicating the periodicity of the kinetic energy changes at waves 2 and 3 is apparently of the order of 4 days.

At wave number 5, there is a marked tendency for a  $\Delta KE$  (5) to be negative at  $t + 12$  hours. A tendency for  $\Delta KE$  (6) to be negative at this same time also exists as shown in Fig. 4c. This suggests a positive correlation between the two. However, the exact nature of any such relationship is difficult to determine by the technique employed here.

At intermediate wave numbers, as represented by  $n = 8$ ,  $r_{8,8}$  tends to be positive at  $t + 12$  hours. At the same

time, however,  $r_{8,6}$  , and  $\bar{r}_{8,10}$  are of opposite sign as may be seen in Fig. 4d. This result points to a general agreement with the results of Saltzman and Fleisher [12], namely, when a decrease of kinetic energy occurs in the cyclone waves, kinetic energy tends to be exchanged both upward and downward in the wave number scale. A schematic illustration of this exchanged energy, after Saltzman and Fleisher [12], is shown in Fig. 5.

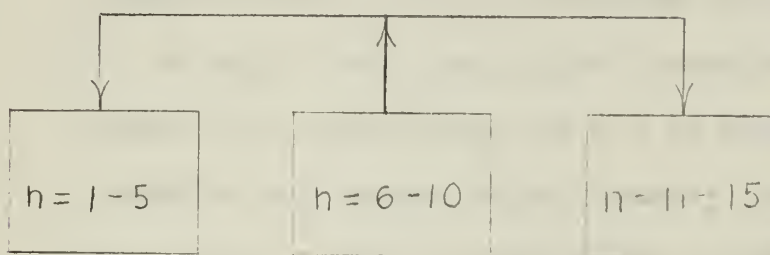


Figure 5. Estimate of the kinetic energy exchange between cyclone waves and longer and shorter waves. After Saltzman and Fleisher [12].

A direct correlation analysis of the kinetic energy changes between the various waves would have yielded seven times as many correlations as the present scheme, so that analysis of the results would be extremely involved. Also the results would have had no relationship to  $T^*(n)$ , which function affords the possibility of being a predictor based on physical considerations.

It is necessary to comment on the relatively small number of significant correlation coefficients. For

example, in Tables 2 and 3, there occurred 27 and 30 significant values, respectively, where 16 (or 10%) could have arisen due to chance. Some of the reasons that a more conclusive number of significant correlations did not appear may be the following:

- (a) Of the several macroscopic eddy stress terms in equation (2.1) only the  $T^*$  term was considered.
- (b) Twenty-four hour changes of KE may not behave linearly when correlated with  $T^*$ .
- (c) In many cases the initial correlation between  $T^*(n)$  and  $\Delta KE(n)$  at  $t - 12$  hours was negative but became significantly positive a day later. This is indicative of the importance of the process of exchange of kinetic energy between the waves in the manner described by Saltzman [12]. Such exchanges undoubtedly contribute to the initial state of the kinetic energy spectrum.
- (d) There is some "noise" present in all Fourier analyses of meteorological parameters. There may also have been some undetectable reproduction errors in the copy of the original data used in this study.

In spite of the shortcomings of the linear correlation technique employed in this study, it is felt that isolating "significant" correlation sequences of the type

shown in Fig. 4, led to several valuable results. Among these are:

- (a) the short period of kinetic energy changes at the high wave numbers;
- (b) the longer period changes at lower wave numbers;
- (c) the tendency of wave number 8 (in the center of the cyclone wave group) to exchange kinetic energy with both waves 6 and 10.

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## APPENDIX

The following tables list the results of correlating the energy transfer function  $T^*(n)$  computed at  $45^\circ\text{N}$ , with the 24-hour kinetic energy change  $\Delta\text{KE}$  computed at  $35^\circ$ ,  $45^\circ$ , and  $55^\circ\text{N}$ . In addition,  $\Delta\text{KE}$  is varied with respect to wave number and time as indicated by the respective table captions. Significant correlation coefficients are denoted by an asterisk.

Central "n"	n - 2	n - 1	n	n + 1	n + 2
1	---	---	.02	-.01	.10
	---	---	-.14	-.05	-.03
	---	---	.30*	.01	.37*
2	---	-.31*	.26	.06	.10
	---	-.01	-.21	.05	-.19
	---	-.10	.06	.03	.00
3	.10	.20	-.10	.08	-.01
	-.01	-.24	.23	.07	.59*
	.07	-.03	.37*	.08	-.09
4	-.29	.04	-.03	-.25	.10
	-.39*	.02	.05	.27	.05
	-.05	.10	-.07	-.21	.01
5	.13	-.06	.08	.10	.15
	.10	-.02	.28	.07	.07
	.00	.12	.05	.08	-.11
6	-.13	.34*	.03	-.54*	-.22
	-.03	.15	.20	-.02	-.03
	.12	.06	.03	-.36*	-.13
7	.21	.26	-.47*	-.35*	.04
	-.08	.06	-.42*	-.28	.37*
	.13	.00	.00	-.10	-.01
8	.40*	-.15	-.27	.23	.28
	.34*	-.22	-.21	.10	.05
	.11	.10	-.23	-.06	-.11
9	.26	-.08	-.02	.00	-.36*
	.15	-.34*	-.36*	-.09	.02
	.22	-.18	-.15	.07	-.11
10	.02	.30*	-.24	-.10	-.19
	.10	-.34*	.33*	.20	.21
	.02	.23	-.08	.18	.08
11	.12	-.13	-.09	.18	---
	.11	.08	.50*	.04	---
	.01	-.19	.61*	.48	---
12	-.09	-.13	.12	---	---
	.31*	.13	.07	---	---
	-.32*	.44*	-.05	---	---

Table 2. Correlations of  $T^*(n)$  at  $45^\circ N$  at time  $t$ , with  $\Delta KE$  at  $n$ ,  $n \pm 1$ , and  $n \pm 2$ , at time  $t - 12$  hours. Significant values are marked with an asterisk.

Central "n"	n - 2	n - 1	n	n + 1	n + 2
1	----	---	.10	.12	-.32
	---	---	.11	.33*	-.19
	---	---	-.03	.23	.00
2	---	-.04	-.05	-.08	-.06
	---	-.08	.36*	-.29	.04
	---	.38*	-.08	-.35*	-.22
3	-.08	-.01	.04	-.37*	-.08
	-.04	-.07	-.39*	-.09	-.28
	.32*	.05	-.24	-.13	-.08
4	-.02	.00	-.09	.09	-.14
	-.14	-.20	-.06	-.38*	-.13
	.12	.20	.06	.03	-.03
5	-.01	.17	.12	-.34*	-.06
	.29	-.07	-.71*	-.16	-.19
	-.10	.07	.04	-.02	-.11
6	.17	-.21	-.10	-.06	-.03
	.06	-.02	-.32*	-.51*	.06
	.05	-.10	.27	-.16	-.52*
7	-.20	-.04	-.05	.21	-.02
	.12	-.07	.11	.38*	.05
	-.21	.15	-.02	-.32*	-.17
8	-.21	.15	.20	-.19	-.62*
	-.43*	-.07	.49	.10	-.08
	.06	.19	-.08	.21	-.15
9	.06	.04	-.43*	-.07	.29
	.01	.11	.39*	-.48*	.02
	.10	.34*	-.12	-.12	.03
10	.08	-.23	.15	-.11	.19
	-.34*	-.07	-.29	-.38*	-.13
	.27	-.19	.27	-.43*	-.42*
11	-.14	.02	.23	-.09	---
	.16	-.36*	-.44*	-.14	---
	-.37*	.25	-.38*	-.27	---
12	-.08	.24	-.17	---	---
	-.15	-.10	.04	---	---
	.16	-.32*	.21	---	---

Table 3. Correlations of  $T^*(n)$  at  $45^{\circ}\text{N}$  and time  $t$ , with  $\Delta(\text{KE})$  at  $n$ ,  $n \pm 1$ , and  $n \pm 2$ , at time  $t + 12$  hours. Significant values are marked with an asterisk.

Central "n"	n - 2	n - 1	n	n + 1	n + 2
1	---	---	-.10	-.06	-.21
	---	---	.19	-.05	.16
	---	---	-.18	-.25	-.35*
2	---	-.14	-.24	.00	-.13
	---	.21	-.09	-.16	-.14
	---	-.13	-.08	.05	.00
3	.36*	-.27	.07	.14	-.01
	.11	.09	.10	-.18	-.05
	-.03	-.29	.23	.35*	.12
4	.33*	.00	.22	.06	-.09
	.29	.18	.04	-.01	.09
	.01	.14	.39*	.04	-.29
5	.05	.02	-.20	.18	.16
	.17	.22	.21	.09	.15
	.07	.03	-.21	.03	-.19
6	-.21	-.17	-.04	-.03	.07
	-.02	-.14	-.24	-.15	-.06
	-.22	.13	.38*	.13	-.12
7	.03	-.10	-.48*	.01	.14
	-.05	-.23	-.08	.01	.03
	.11	.14	-.04	-.20	.30*
8	-.07	-.44*	.22	.15	.44*
	-.08	.14	-.06	-.06	.00
	-.07	-.13	.12	.07	.20
9	.17	-.03	.41*	.18	-.01
	.16	.23	-.02	.58*	.00
	.07	-.04	.11	.06	.15
10	-.29	-.03	-.23	.11	.03
	.08	.19	.06	.22	.05
	-.00	.09	-.32*	.07	.19
11	.18	.08	-.08	-.18	---
	-.09	.35*	.38*	.17	---
	.41*	-.14	.28	.02	---
12	.14	-.04	-.09	---	---
	-.11	.11	-.15	---	---
	-.03	.08	-.15	---	---

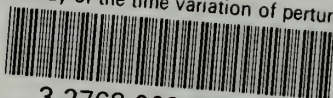
Table 4. Correlations of  $T^*(n)$  at  $45^{\circ}\text{N}$  and time  $t$  with  $\Delta KE$  at  $n$ ,  $n \pm 1$  and  $n \pm 2$ , at time  $t - 36$  hours. Significant values are marked with an asterisk.

Central "n"	n - 2	n - 1	n	n + 1	n + 2
1	---	---	.20	.05	.39*
	---	---	-.04	-.28	.07
	---	---	-.02	.03	.08
2	---	.06	-.05	.08	.13
	---	-.29	.34*	-.20	.17
	---	.10	-.03	-.09	-.24
3	-.42*	.24	-.05	.15	.06
	-.06	.13	.17	.42*	.05
	-.05	.46*	-.47*	-.15	.00
4	.02	-.06	-.05	.09	.14
	-.30*	-.17	.31*	.05	.13
	.36*	-.04	-.04	.05	.26
5	-.01	-.07	.09	.10	-.04
	-.57*	.25	-.05	-.05	.45*
	.18	-.15	.02	.13	.14
6	.07	-.09	.13	-.03	-.05
	.04	.06	.11	.11	.03
	-.03	.04	-.41*	.23	.05
7	-.02	-.14	.41	-.03	.08
	.02	.04	-.33*	-.10	-.43*
	.26	-.03	.03	.02	-.20
8	.04	.35*	-.17	.11	.11
	.21	-.20	.00	-.43*	-.09
	-.09	-.26	-.04	-.14	-.08
9	-.19	.09	.01	-.05	-.25
	.04	-.13	-.18	.03	.13
	-.27	-.29	-.01	.10	-.06
10	.16	-.22	-.07	.00	.01
	-.11	.05	-.10	-.23	-.11
	-.06	.05	.26	.00	.08
11	-.12	-.02	-.10	.14	---
	-.31*	-.30*	-.28	-.07	---
	-.31*	.18	-.17	.09	---
12	-.04	.04	.14	---	---
	-.12	-.08	.00	---	---
	.17	.01	.19	---	---

Table 5. Correlations of  $T^*(n)$  at  $45^{\circ}\text{N}$  and time  $t$ , with  $\Delta\text{KE}$  at  $n$ ,  $n \pm 1$ , and  $n \pm 2$ , at time  $t + 60$  hours. Significant values are marked with an asterisk.

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